14.4 More Applications

1. Cost Breakdown (HW 14.3/1-2)

Suppose the cost to produce ONE item is given by:

$$C(x, y) = 3x^2 + 4y^2 + 5xy + 10,$$

where

$x =$ cost for 1 hour of labor, and
$y =$ cost for 1 pound of materials.

Entry Task:

The current hourly rate for labor is \$20 and material is \$55 per pound.
How will a \$1 per hour raise for labor affect the cost to produce 1 item?

$C_x = 6x + 5y \quad \frac{\text{dollars}}{\text{labor hour}}$

$C_y = 8y + 5x \quad \frac{\text{dollars}}{\text{pound}}$

$C_x(20, 55) = 6(20) + 5(55)$

$= \boxed{395 \ \$/\text{labor hour}}$

IF LABOR COSTS GO UP \$1/hr THEN IT WILL COST \$395 MORE TO PRODUCE ONE ITEM.

2. Marginal Productivity (14.3/5-6)

Suppose that the number of crates of a particular fruit produced is

$$z = \frac{9xy - 0.0002x^2 - 5y}{0.03x + 4y} \begin{matrix} \leftarrow N \\ \leftarrow D \end{matrix}$$

where

$x =$ number of hours of labor, and
$y =$ number of acres of the crop.

Find the marginal productivity for hours of labor when x = 100 and y = 200. Interpret your answer

$$N = 9xy - 0.0002x^2 - 5y$$

$$N_x = 9y - 0.0004x$$

$$D = 0.03x + 4y$$

$$D_x = 0.03$$

$$\frac{\partial z}{\partial x} = \frac{DN' - ND'}{D^2}$$

$$= \frac{(0.03x+4y)(9y-0.0004x) - (9xy - 0.0002x^2 - 5y)(0.03)}{(0.03x+4y)^2}$$

AT $x = 100$, $y = 200$

$$\frac{\partial z}{\partial x} = \frac{(803)(1799.96) - (178998)(0.03)}{(803)^2}$$

$$\frac{\partial z}{\partial x} = \frac{\text{crates produced}}{\text{labor hours}} = \text{MARGINAL PRODUCTIVITY} \approx 2.23322 \ \frac{\text{crates}}{\text{labor hour}}$$

IF LABOR HOURS IS INCREASED BY ONE THEN ABOUT 2.23 more crates of fruit will be produced.

3. Revenue/Cost

Assume you manufacture and sell two products, *A* and *B*.

Let

$x =$ thousands of units of A, and

$y =$ thousands of units of B.

You know from past years that your cost (in thousand dollars) is given by

$C(x, y) = 2x^2 - 2xy + y^2 - 9x - 10y + 11$

And you know:

- Product A sells for \$5.00/item, and
- Product B sells for \$8.00/item.

What is the maximum profit?

$$R(x,y) = 5x + 8y$$

REVENUE!

$P(x,y) =$ PROFIT

$= R(x,y) - C(x,y)$

$= [5x + 8y] - [2x^2 - 2xy + y^2 - 9x - 10y + 11]$

$= 5x + 8y - 2x^2 + 2xy - y^2 + 9x + 10y - 11$

$\boxed{P(x,y) = -2x^2 + 2xy - y^2 + 14x + 18y - 11}$

$P_x = -4x + 2y + 14 \overset{?}{=} 0 \Rightarrow 2y = 4x - 14$

$y = 2x - 7$

$P_y = 2x - 2y + 18 \overset{?}{=} 0$

$2x - 2(2x - 7) + 18 \overset{?}{=} 0$

$2x - 4x + 14 + 18 \overset{?}{=} 0$

$32 = 2x$

$\boxed{\begin{array}{l} x = 16 \\ y = 2(16) - 7 = 25 \end{array}}$

$\boxed{\begin{array}{l} P(16,25) = -2(16)^2 + 2(16)(25) - (25)^2 \\ \quad + 14(16) + 18(25) - 11 \\ = 326 \end{array}}$

PROFIT IS MAXIMIZED WHEN WE PRODUCE AND SELL 16 thousand of product A, 25 thousand of product B, and MAX PROFIT IS $\boxed{\text{326 thousand dollars}}$